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Design of a Reinforced Concrete Arch

Civil Engineering

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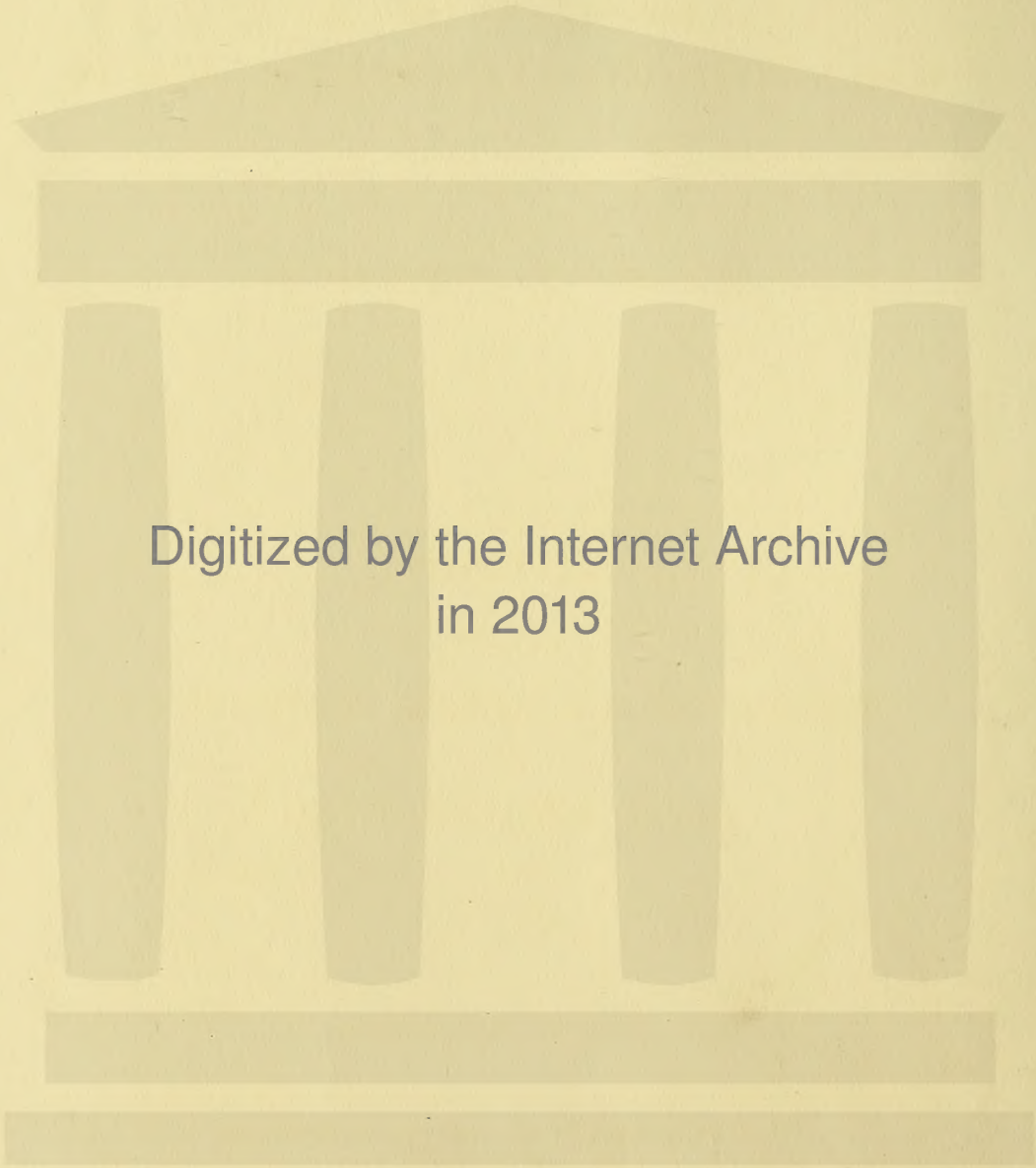

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DESIGN OF A REINFORCED CONCRETE ARCH

BY

HOWARD STAAT MALCOLM

THESIS

FOR THE

DEGREE OF BACHELOR OF SCIENCE

IN

CIVIL ENGINEERING

COLLEGE OF ENGINEERING

UNIVERSITY OF ILLINOIS

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June 1, 1908

THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

HOWARD STAAT MALCOLM

ENTITLED DESIGN OF A REINFORCED-CONCRETE ARCH

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Bachelor of Science in Civil Engineering

J. J. Richey
Instructor in Charge.

APPROVED: Ira O. Baker.

HEAD OF DEPARTMENT OF Civil Engineering

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Design of a Reinforced Concrete Arch.

The arch here designed is intended for a city railroad crossing. While no definite location is taken, it is assumed that the span is 80 feet, and the maximum clearance obtainable for rise is 11 feet on account of existing grades. This small rise leads to some boldness of design, but an examination of existing highway arches leads to the conclusion that if heavily reinforced, such an arch will safely carry the heaviest loads, namely Cooper's Class E50.

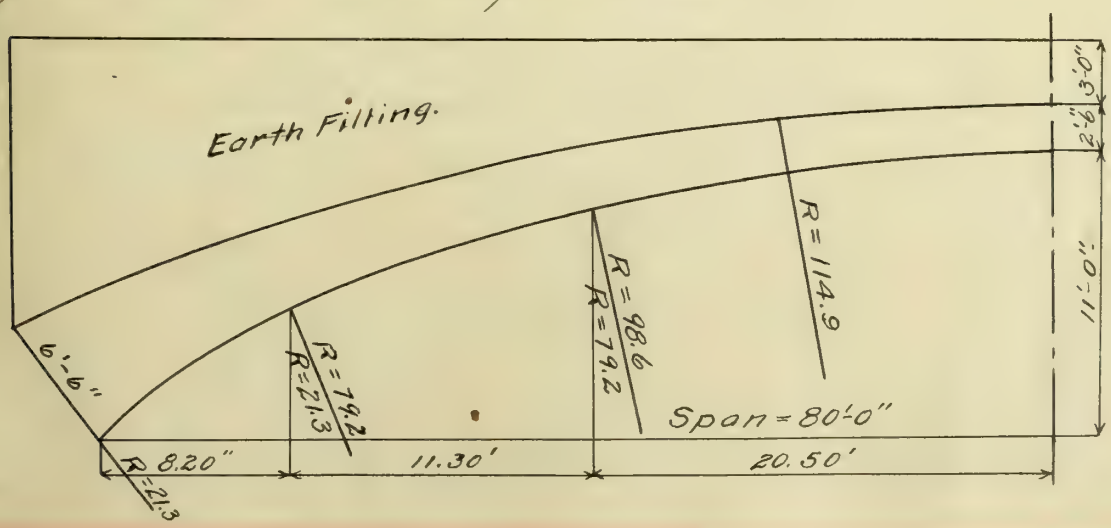
The Melan type of reinforcement consisting of four angles latticed together was adopted because the web enables the steel frame to be self supporting and to carry shearing stress, and hence makes a more rigid arch than could be secured by the use of steel bars. If the latter were used such heavy reinforcement as here necessary would require the use of heavy bars placed close together, which would be objectionable on account of the difficulty of placing the concrete. Lattice ribs also furnish

UIUC

a good bond with the concrete

Prof. Cain's graphical solution of the solid arch as given in his "Theory of Steel-Concrete Arches," has been used in the design. It is believed that Cain's theory affords as simple and as correct a solution as any, and was selected because graphical methods could be so largely employed.

All the dimensions and assumptions necessary for the design are given together below, so that they can be referred to readily. The allowable stresses and the assumed weights of the materials are those which have been generally accepted. The dimensions were assumed after a careful study of existing arches described in the different engineering magazines, and the table of arches given in Prof. Howe's work on "Symmetrical Masonry Arches"



Dimensions:

Fill over the crown, 3 feet of earth.

Ties, 8 feet long

Width of arch, 32 feet.

Live load assumed to be distributed over a width of arch equal to 13 feet.

Reinforcement:

20 ribs 18½ inches center to center, placed 2½ inches from the extrados and intrados, and the outside flange of the angles coming within 3 inches of the sides of the arch. Each rib is composed of 4 Ls 4"x4"x9/16", net area of each angle, subtracting for 2-¾" rivets for splice, equals 3.28 square inches.

Lattice bars, 2"x⅜" placed at an angle of 45°.

Percentage of reinforcement at the crown equals 2.80%; at springing line, about 1%

Loads:

Live load 7000 pounds per linear foot or 7000/13 = 540 pounds per square foot.

Weight of track 450 pounds per linear foot or 35 pounds per square foot.

Weight of Materials:

Weight of concrete and steel 150 pounds per cu. ft.

Weight of earth filling 100 pounds per sq. ft.

Modulus of Elasticity:

Concrete = 1,400,000

Steel = 28000000

Allowable Stresses:

Compression on 1:2:4 concrete = 500 pounds per square foot.

Tension or compression in steel = 15000 pounds per square inch.

Division of Neutral Axis:

Nomenclature,

$d, d_1, d_2 \ldots$ = depth of arch ring at different points

$S, S_1, S_2 \ldots$ = Lengths of the divisions of the neutral axis

$I_1 = \frac{d_1^3}{12}$ = moment of inertia of concrete at a section

I_2 = moment of inertia of steel at a section.

$n = \frac{\text{modulus of elasticity of steel}}{\text{modulus of elasticity of concrete}} = 20.$

$l.$ = distance along the neutral axis from springing line to center of S.

The neutral axis was divided into lengths, so that each length divided by the moment of inertia, $\frac{S}{I_1 + nI_2}$, is constant for all

divisions, since by so dividing the computation of stress is very much simplified. Here

$$S \div (I_1 + nI_2) = S \div \left[\tfrac{1}{12}d^3 + nA_2\left(\tfrac{d}{2} - x\right)\right].$$

A_2 = area of steel at a section = $\frac{13.12}{1.541}$ square inches = 0.059 square feet.

$nA_2 = 20 \times 0.059 = 1.18$ square feet.

x = distance of the steel from the intrados and extrados in feet.

Thus in order that $S \div (I_1 + nI_2)$ be constant,

$$\frac{S}{\frac{1}{12}d^3 + 1.18\left(\frac{d}{2} - 0.19\right)^2} = \frac{S_1}{\frac{1}{12}d^3 + 1.18\left(\frac{d}{2} - 0.19\right)^2}$$

In dividing the neutral axis the length of the first division was assumed. This reduced the equation for S to $S = k\left[\frac{d^3}{12} + 1.18\left(\frac{d}{2} - 0.19\right)^2\right]$, and fixed the lengths of the remaining divisions. In order to get a convenient number of divisions, and in order that their sum equal the length of the neutral axis the assumed length of the first division was slightly changed until these conditions were obtained. Upon the third trial the sum of the divisions was 0.08 feet in excess of the length of the neutral axis. This was distributed among the divisions by comparing the corresponding values of S in the second and third trials. The final division gave $k = 0.934$.

In making the trial divisions it was

found convenient to find I and compute $\frac{1}{12}d^3 + 1.18\left(\frac{d}{2} - 0.19\right)^2$ for values of d along the neutral axis. A curve was drawn with values of I as abscissæ and values of $\frac{1}{12}d^3 + 1.18\left(\frac{d}{2} - 0.19\right)^2$ as ordinates. Values of S were then found which gave $S = k\left[\frac{d^3}{2} + 1.18\left(\frac{d}{2} - 0.19\right)^2\right]$, by finding I and taking the value of $\frac{1}{12}d^3 + 1.18\left(\frac{d}{2} - 0.19\right)^2$ from the curve.

The final values of S are given in Table I.

Five lengths were laid off in succession on the neutral axis from both springing lines to the crown. The centers of the divisions were found and marked $a_1, a_2 \cdots a_{24}$ as shown in Plate I.

Dead Load: Vertical lines were drawn through $a_1, a_2 \cdots a_{24}$, and the areas included between the successive verticals, the intrados, and the upper limit of the reduced filling, found. The filling was reduced by assuming it to be replaced by an equal weight of concrete. These areas were multiplied by 150 which gives the dead load for each division.

Live Load: Since the live load is transmitted from the wheels to the arch ring by the earth, the load was considered as uniformly

TABLE I

VALUES OF S.

No.	Value of S Ft.	Value of l Ft.	No.	Value of S Ft.	Value of l Ft.
S_1	10.67	5.34	S_7	2.72	29.89
S_2	4.77	13.05	S_8	2.68	32.59
S_3	3.87	17.38	S_9	2.64	35.25
S_4	3.36	20.99	S_{10}	2.59	37.86
S_5	3.04	24.19	S_{11}	2.54	40.43
S_6	2.82	27.12	S_{12}	2.50	42.95

distributed. The live load, as well as the weight of the track was found for each division by multiplying the horizontal length of each division by the load per square foot, or $l \times (540 + 35)$. It was assumed that the live load produced its maximum effect when it covered one half of the arch.

Horizontal Pressure of Filling: The theory employed does not take into consideration the horizontal pressure of the filling. This pressure was computed by the equation given in Prof. Baker's book on Masonry, $h = \frac{1}{3} c d l$, in which

h = horizontal pressure due to earth filling and live load reduced to equivalent earth filling.

c = weight of filling per cubic foot.

d = depth of center of pressed surface below top of filling.

l = vertical dimension of the pressed surface

The total amount of this force due to filling and live load is 3900 pounds, of which 1400 pounds acts on the division next to the springing line. The small amount of this force showed that it could be safely neglected.

Table II gives the total load for each division. The total load is designated by $P_0, P_1 \cdots P_{25}$

TABLE II
LOADS.

No. of Loads	Sq. Feet in Arch Ring & Reduced Filling.	Dead Load Lbs.	Live Load Lbs.	Combined Dead and Live Load Lbs.
P_0	77.94	11690		11690
P_1	67.68	10150		10150
P_2	30.96	4640		4640
P_3	23.85	3580		3580
P_4	19.08	2860		2860
P_5	16.74	2510		2510
P_6	15.21	2280		2280
P_7	13.59	2040		2040
P_8	12.78	1920		1920
P_9	12.15	1820		1820
P_{10}	11.61	1740		1740
P_{11}	11.34	1700		1700
P_{12}	5.67	850		850
P_{13}	5.67	850	690	1540
P_{14}	11.34	1700	1430	3130
P_{15}	11.61	1740	1470	3210
P_{16}	12.15	1820	1490	3310
P_{17}	12.78	1920	1510	3430
P_{18}	13.59	2040	1530	3570
P_{19}	15.21	2280	1580	3860
P_{20}	16.74	2510	1660	4160
P_{21}	19.08	2860	1820	4680
P_{22}	23.85	3580	2040	5620
P_{23}	30.96	4640	2380	7120
P_{24}	67.68	10150	4120	14270
P_{25}	77.94	11690	4010	15700
	Totals	95580	25850	121430

The loads $P_1 \cdots P_{24}$ were laid off at the center of gravity of the divisions between the verticals $a_1, a_2, \cdots a_{24}$, as shown in Plate I. The loads were also laid off on the vertical load line to the left of the arch to the scale of loads. A pole distance of 40000 pounds was then assumed and this horizontal thrust laid off to the right on a horizontal line through the lower end of P_{12}.

Equilibrium Polygon: The equilibrium polygon $b_1, b_2 \cdots b_{24}$ was then constructed, and the closing line $m m_1$ found by the usual method. The work necessary for doing this is given below, but for explanation of the method the reader is referred to Cain's "Theory of Steel-Concrete Arches."

The line $v_1 v_{24}$ was drawn, and $R = (v_2 b_2 + v_3 b_3 + \cdots v_{23} b_{23})$ or the sum of the ordinates from $v_1 v_{20}$ to $b_2, b_3 \cdots b_{23}$, found to be 347.9, to the scale of distance. These ordinates were then treated as forces and the position of this resultant, R, found by taking moments about AB. Trial T_1 or the sum of the ordinates through $b_1, b_2 \cdots b_{23}$ from $v_1 v_{24}$ to $v_1 n$, was found to equal 197.9 to the scale of distance, and Trial T_1, the sum of the ordinates from $n v_1$ to $n m_1$, to equal 164.4. This

position with regard to AB was found in a manner similar to that for R. R was found to act $\frac{347.9}{345}$ or 1.01 feet to the left of AB. Trial T was found to act $\frac{204.7}{197.9} = 10.42$ feet to the left of AB, and Trial T_1, the same distance to the right.

Taking moments about Trial T_1 and Trial T in turn,

$$\text{True } T = \frac{348 \times 11.43}{20.84} = 191.3.$$

$$\text{True } T_1 = \frac{348 \times 9.41}{20.84} = 157.2$$

Hence in locating m m,

$$v_{24}\, m = \frac{\text{True } T}{\text{Trial } T}\, v_{24}\, M = \frac{191.3}{198.0} \times 16.47 = 15.92,$$

and

$$v_1\, m = \frac{\text{True } T_1}{\text{Trial } T_1} = v_1\, m_1 = \frac{157.2}{164.4} \times 13.75 = 13.18$$

A line connecting the centers of the springing joints, OO_1, was drawn. The line $k k_1$ was then located by measuring the ordinates $y_1 \cdots y_{24}$ from OO_1 to $a_1, a_2, \cdots a_{24}$, and dividing by the number of ordinates giving the distance e of k above OO_1. e was found to equal 0.19 feet.

Next was found $\Sigma(k a \cdot y)$ for the entire arch. This is equal to $\Sigma y^2 - e \Sigma y = 2(858.67 - 804.83) = 107.68$. A similar sum $\Sigma(m b \cdot y)$ was also found for the entire arch. This summation may be written

$$\Sigma(m b \cdot y) = +(m b_{12} + m b_{13}) y_{12} + (m b_{11} + m b_{14}) y_{11} +$$
$$(m b_{10} + m b_{15}) y_{10} + (m b_9 + m b_{16}) y_9 + (m b_8 + m b_{17}) y_8 +$$

$(mb_7 + mb_{18})\,y_7 + (mb_6 + mb_{19})\,y_6 + (mb_{20} - mb_5)\,y_5 -$
$(mb_4 + mb_{21})\,y_4 - (mb_3 + mb_{22})\,y_3 - (mb_2 + mb_{23})\,y_2 -$
$(mb_1 + mb_{24})\,y_1 = 524.33 - 235.96 = 288.37.$

The ordinates mb were altered in the ratio of $\frac{107.68}{288.27} = \frac{1}{2.68}$ and laid off above or below k k, in the same vertical with $a_1, a_2, \ldots a_{24}$, thus locating the points $c_1 \ldots c_{24}$. The pole distance was altered in the ratio of 2.68 : 1. To find the true pole, a line parallel to m m, was drawn from the trial pole. Where this line intersected the load line a horizontal line was drawn to the right. On this line was laid off a distance equal to $40000 \times 2.68 = 107200$ pounds, the true horizontal thrust, and the true pole located. The equilibrium polygon $c_1, c_2, \ldots c_{24}$ was thus checked by the usual method.

Unit Stresses: The unit stresses due to dead and live load at the several divisions were computed according to the equations,

$$S_1 = \frac{T}{A_1 + nA_2} \pm \frac{M v_1}{\frac{1}{12} d_1^2 + nA_2 v_2^2} \text{ for concrete}$$

$$S_2 = \left(\frac{T}{A + nA_2} \pm \frac{M v_2}{\frac{1}{12} d_1^2 + nA_2 v_2^2}\right) n \text{ for steel.}$$

Here T = normal component of the thrust at a section in pounds.

A_1 = area of concrete in square feet.

A_2 = area of steel in square feet.

M = moment due to the thrust at a section,

or H multiplied by its distance from the neutral axis.

v_1 = one half the depth of the arch rng.

v_2 = distance from neutral axis to the centre of gravity of the flange.

Only those stresses which, combined with temperature stresses, give maximum values are shown below.

Point a_1.

$$S_1 = \frac{115200}{4.35+1.18} \pm \frac{107100 \times 0.47 \times 2.17}{6.91+1.18+3.92}$$

$= 20900 \pm 9300$ pounds per square foot.

$S_1 = 146 \pm 65$ pounds per square inch.

$= 211$ pounds compression at the extrados and 81 pounds compression at the intrados.

Similarly,

$S_2 = 2140$ pounds per square inch compression at the extrados and 1700 pounds compression at the intrados.

Point a_9.

$$S_1 = \frac{107700}{2.55+1.18} \pm \frac{107100 \times 0.22 \times 1.27}{1.38+1.18 \times 1.17}$$

$= 28900 \pm 10880$ pounds per square foot.

$S_1 = 200 \pm 76$ pounds per square inch.

$= 276$ pounds compression at the intrados and 134 pounds compression at the extrados.

$S_2 = 5400$ pounds compression at the intrados

and 2730 pounds compression at the extrados.

Temperature Stresses: The thrust due to temperature is given by the equation

$$H = \frac{E l \epsilon t^\circ}{(\Sigma y^2 - \frac{1}{l}\Sigma y)} \times \frac{I_1 + n I_2}{S}.$$

In which,

l = Length of the neutral axis.

$\epsilon = 0.000012$, the coefficient of expansion of concrete per Celsius degree.

t° = rise or fall of temperature in Celsius degrees above or below the mean temperature.

(1 Celsius degree = 1.8°F.).

Just what rise and fall of temperature to assume is a matter of doubt. Concrete is a very poor conductor of heat. Moreover the extrados is covered with earth so that its top cannot be subjected to any great extremes of temperature, while the intrados is shaded by the arch itself. In the computations a fall of 43°F and a rise of 27°F from the mean temperature of 53°F was taken. This gives the extremes of temperature as 10°F and 80°F, which are probably greater than would exist in such an arch in a temperate climate.

Substituting in the equation above,

$$H = \frac{1400000 \times 144 \times 98.40 \times 0.000012 \times \frac{43}{1.8}}{107.68} \times \frac{1}{0.934}$$

= 50100 pounds, corresponding to a fall of 43°F.

The stresses due to this thrust are computed by the same equations as those used in computing the stresses caused by dead and live load. T is here the component of the thrust perpendicular to the face of the section, and $M = H \cdot kQ$, in which kQ is the perpendicular distance from Q, to kk_1.

The temperature stresses at a_1 and a_9 will now be computed.

Point a_1. For a fall in temperature.

$$S_1 = \frac{45800}{5.53} \pm \frac{50100 \times 5.40 \times 2.17}{11.53}$$

$S_1 = 60 \pm 347$ pounds per square inch.

= 407 pounds compression at the intrados and 287 pounds tension at the extrados.

$S_2 = 8020$ pounds compression at the intrados and 5880 pounds tension at the extrados.

For a rise in temperature: The temperature stresses in this case are $\frac{27}{43}$ as much as for a fall of temperature, but the expansion of the concrete causes tension at the intrados and compression at the extrados when the center of gravity of the section is below kk_1 and vice versa when it is above.

Hence $S_1 = 407 \times \frac{27}{43} = 252$ pounds compression at extrados and 173 pounds tension at the intrados

$S_2 = 4920$ pounds compression at the extrados and 3580 pounds tension at the intrados

Point a_9. For a fall of temperature.

$$S_1 = \frac{49900}{3.73} \pm \frac{50400 \times 1.51 \times 1.27}{2.76}$$

$S_1 = 92 \pm 243$ pounds per square inch.

= 335 pounds compression at the extrados and 151 pounds tension at the intrados.

S_2 = 6570 pounds compression at the extrados and 3160 pounds tension at the intrados.

For a rise of temperature.

S_1 = 211 pounds compression at the intrados and 95 pounds tension at the extrados.

S_2 = 4120 pounds compression at the intrados and 1980 pounds tension at the extrados.

These stresses have been combined with the dead and live load stresses in Table III. It is seen that the maximum compressive stresses at a_1 and a_9 are the same.

Since the 92 pounds tension is not excessive for temperature stresses, and occurs only under the extreme temperature, a modification of the arch was not considered necessary. Concrete of a proportion of 1:2:4 is easily capable of resisting 200 pounds tension, which gives a factor of safety of over two

Assumption that steel resists entire bending moment due to temperature.

Point a_1

Maximum compression in lower fibre due to live load and dead load equals 1700 $\frac{\text{pounds}}{\text{square inch}}$

Maximum compression in upper fibre due to live load and dead load equals 4140 $\frac{\text{pounds}}{\text{square inch}}$.

Moment due to $\pm 43°F = H \cdot ha = 50100 \times 54 = 270500$ foot pounds.

Area of stud = 4.25 square inches.

Distance center to center of stud = 3.78 feet.

Total stress in stud = $\frac{270500}{3.78} = 73300$ pounds.

Stress in pounds per square inch = $\frac{73300}{4.25} =$ 17300.

Maximum compression = $-17300 - 4140 = -21440$ pounds per square inch.

Maximum tension = $+17300 - 1700 = 15600$ pounds per square inch.

This shows that if the arch ring should crack entirely through at a_1, the stud would carry the maximum stresses due to temperature even when combined with the dead and live load stresses, and the total stress still be well within the elastic limit. It will be noticed that the stress in the stud is for a rise or fall of 43°F, which allows for a greater rise of temperature than would be probable.

TABLE III

MAX. & MIN. STRESSES.

Section	Concrete or Steel	Extrados or Intrados	Stress due to D.L & L.L. Lbs per Sq In	Stress due to rise of 27°F. Lbs per Sq In	Stress due to fall of 43°F. Lbs per Sq. In	Max. stress at 80°F. Lbs per Sq. In	Max. stress at 10°F Lbs per Sq In
a_1	Concrete	Extrados	-211	-252	+287	-463	+76
"	"	Intrados	-81	+173	-407	+92	-488
"	Steel	Extrados	-4140	-4920	+5880	-9060	+1740
"	"	Intrados	-1700	+3580	-8020	+1880	-9720
a_9	Concrete	Extrados	-134	+95	-335	-39	-469
"	"	Intrados	-276	-211	+151	-487	-125
"	Steel	Extrados	-2730	+1980	-6510	-750	-9240
"	"	Intrados	-5400	-4120	+3160	-9520	-2240

Note:

− = Compression.

+ = Tension.

Conclusion: The preceding analysis shows that a reinforced arch of small rise & economical section can be constructed. In the analysis the line of resistance is found to pass everywhere within the middle third of the sections, and through the centre of the crown. The stresses due to dead load and live load are low and could not be materially decreased by increasing the rise on account of the fact that the weight of the structure would be increased. The crown thrust due to temperature is considerably greater than in arches of greater rise, but the bending moment is not so much greater because the moment arm, $k a_1$ is shorter. However, the temperature stresses are still somewhat greater than for a higher arch, and the percentage of reinforcement could be decreased slightly if the rise were increased.

The elastic theory is the best guide in designing reinforced concrete arches and may be confidently used when no tension due to dead load and live load is developed. However, the theories concerning the stresses due to changes of temperature are in an unsatisfactory condition. There is doubt as to what tension in the concrete to allow. The number of arches

of small size & very light section which stand without cracking seems to indicate that temperature changes produce no such stresses as computed.

There are two other considerations, shrinkage of the concrete, and permanent set under loading, which modify the stresses as given in Table III. The shrinkage has the effect of decreasing the stress in the concrete and adding it to the steel. Permanent set has the same effect as a fall of temperature. Since the effect of these is not great they have been neglected.

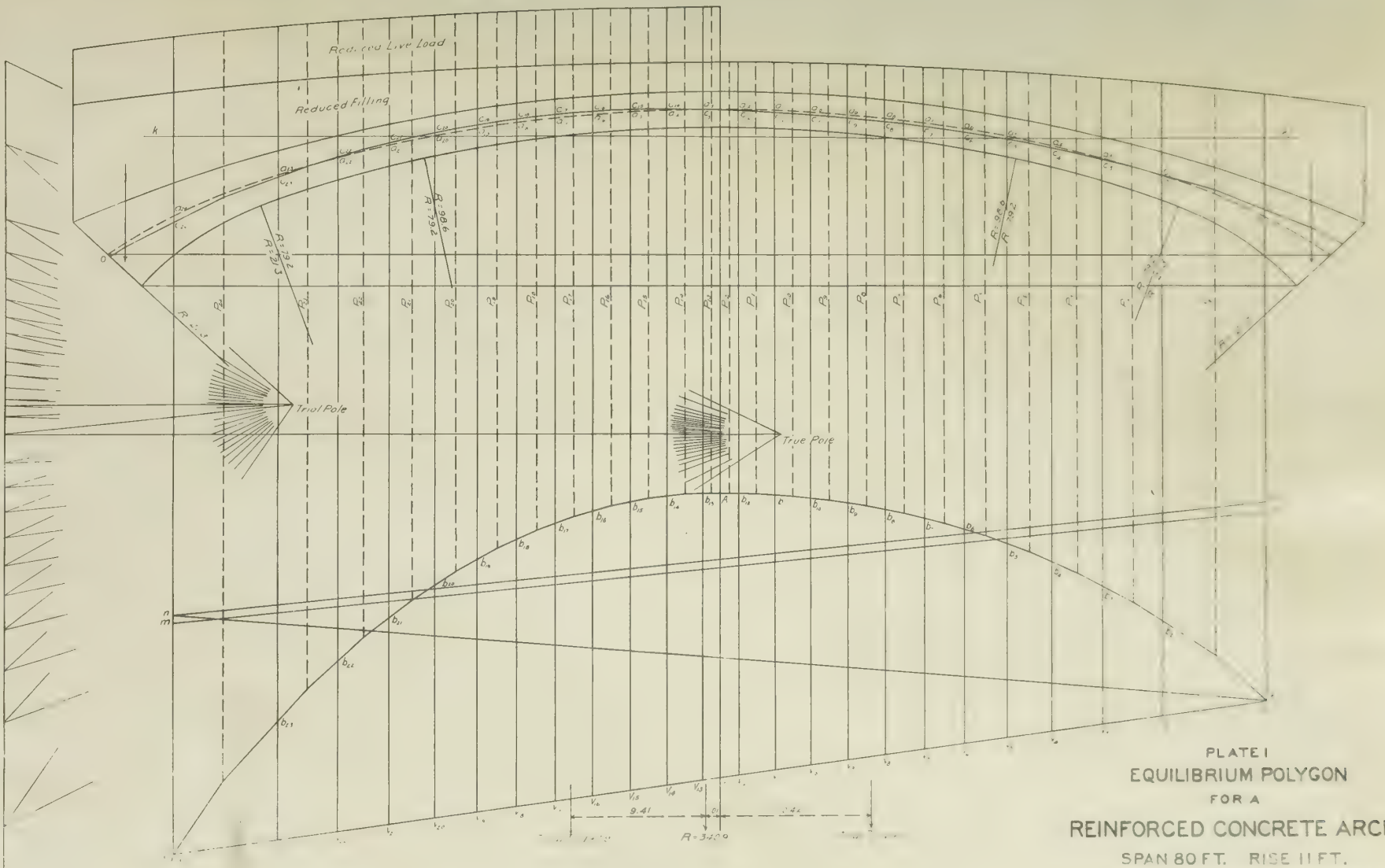


PLATE I

EQUILIBRIUM POLYGON

FOR A

REINFORCED CONCRETE ARCH.

SPAN 80 FT. RISE 11 FT.

SCALES: 1 IN = 10.5 FT. & 1 IN. = 21000 LBS

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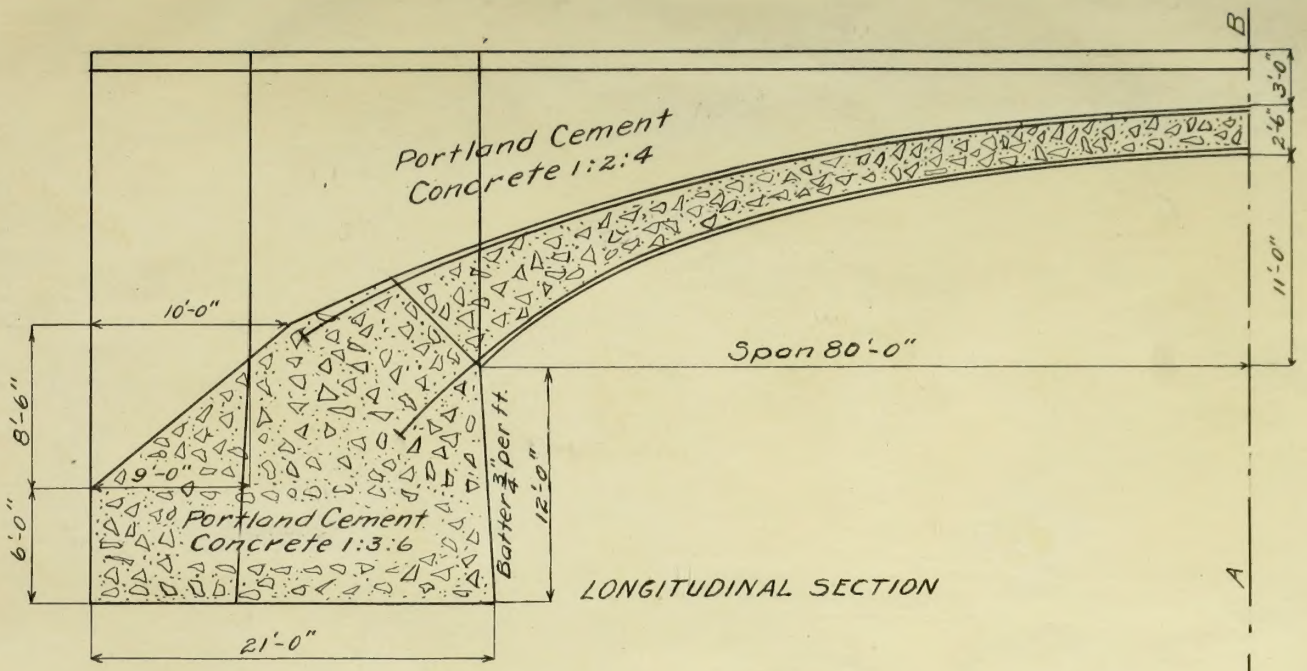
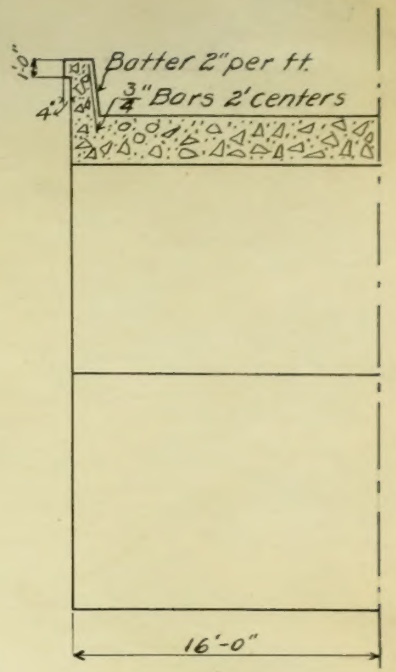
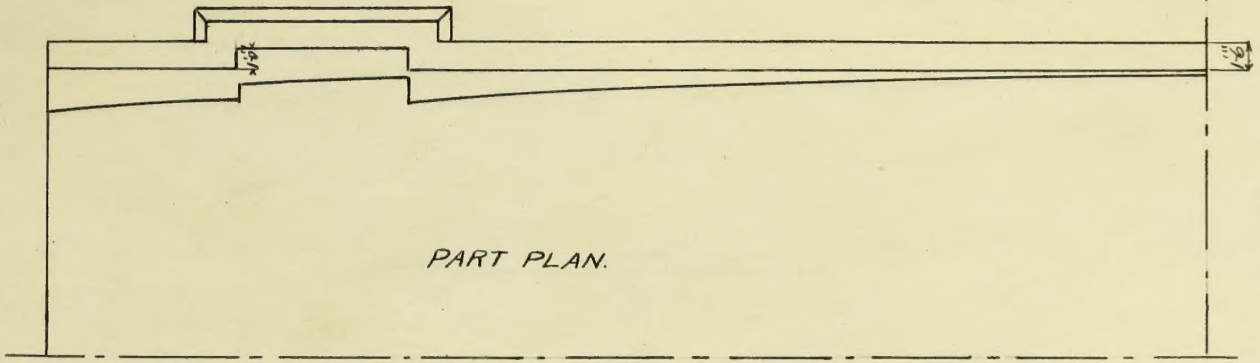




SECTION A-B



PART PLAN.

PLATE II.

SPANDREL WALLS.

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